

**NEP CBCS: 2023-2024**

**M.Sc-II**

**Mathematics**



**M. C. E. Society's**

**Abeda Inamdar Senior College**

Of Arts, Science and Commerce, Camp, Pune-1

(Autonomous) Affiliated to SavitribaiPhule Pune University

NAAC accredited 'A' Grade

**Two Year M.Sc. Degree Program in Mathematics  
(Faculty of Science & Technology)**

**Syllabus for**

**M.Sc.-II (Mathematics)**

**Choice Based Credit System Syllabus**

**To be implemented from the academic year 2024-2025**

**Title of the Course: M.Sc (Mathematics)****Aims and Objectives of the Course**

<b>Sr. No.</b>	<b>Objectives</b>
1.	To maintain an updated curriculum.
2.	To take care of fast development in the knowledge of mathematics
3.	To enhance the quality and standards of Mathematics Education.
4.	To provide a broad common framework, for exchange, mobility, and free dialogue across the Indian Mathematical and associated community.

**Expected Course Specific Learning Outcome**

<b>Sr. No.</b>	<b>Objectives</b>
1.	Students will have an aptitude to Study higher Mathematics and creative work in Mathematics.
2.	Students will equipped themselves with that part of Mathematics which is needed for various branches of Sciences or Humanities in which they have an aptitude for higher studies and original work.

**Structure of M.Sc-I Mathematics Course**

<b>Sr. No.</b>	<b>Major Mandatory</b>		<b>Continuous Internal Evaluation (CIE) (Internal Marks)</b>	<b>End Semester Exam (External Marks)</b>	<b>Total Marks</b>	<b>Credits</b>
	<b>Semester-I</b>	<b>Semester-II</b>				
1.	23SMMT11MM: Linear Algebra	23SMMT21MM: Advanced Calculus	50	50	100	04
2.	23SMMT12MM: Real Analysis	23SMMT22MM: General Topology	50	50	100	04

3.	23SMMT13MM: Group Theory	23SMMT23MM: Rings and Modules	50	50	100	04
4.	23SMMT14MM: Ordinary Differential Equations	23SMMT24MM: Partial Differential Equations	25	25	50	02
5.	23SMMT11RM: Research Methodology	23SMMT21OJ: OJT/FP			100	04
<b>Major Electives (Any One)</b>						
6.	23SMMT11MEA: Discrete Mathematics	23SMMT21MEA: Coding Theory	50	50	100	04
7.	23SMMT11MEB: Advanced Numerical Analysis	23SMMT21MEB: Integral Equations	50	50	100	04
8.	23SMMT11MEC: course from Swayam /NPTEL / E- Pathashala etc.	23SMMT21MEC: course from Swayam /NPTEL / E- Pathashala etc.			100	04

### Structure of M.Sc-II Mathematics Course

Sr. No.	Major Mandatory		Continuous Internal Evaluation (CIE) (Internal Marks)	End Semester Exam (External Marks)	Total Marks	Credits
	Semester-III	Semester-IV				
1.	23SMMT31MM: Complex Analysis	23SMMT41MM: Functional Analysis	50	50	100	04
2.	23SMMT32MM: Field Theory	23SMMT42MM: Differential Geometry	50	50	100	04
3.	23SMMT33TMM: Programming with Python (Theory)	23SMMT43TMM: Introduction to Data Science (Theory)	25	25	50	02

4.	23SMMT33PMM: Programming with Python (Practical)	23SMMT43PMM: Introduction to Data Science (Practical)	25	25	50	02
5.	23SMMT34MM: Fourier Series and Boundary Value Problems	-	25	25	50	02
6.	23SMMT31RP: Research Project	-			100	04
	-	23SMMT41RP: Research Project			150	06
<b>Major Electives (Any One )</b>						
7.	23SMMT31MEA: Probability and Stochastic Process	23SMMT41MEA: Number Theory	50	50	100	04
8.	23SMMT31MEB: Mechanics	23SMMT41MEB: Statistical Inference	50	50	100	04
9.	23SMMT31MEC: course from Swayam /NPTEL / E- Pathashala etc.	23SMMT41MEC: course from Swayam /NPTEL / E- Pathashala etc.			100	04

**For Continuous Internal Evaluation (CIE)**, evaluation of theory courses will be done continuously throughout the semester. CIE will be of 50% marks for CGPA papers.

**CIE for 4 credits theory paper:** It will be divided as follows:

<b>SR. NO.</b>	<b>COMPONENTS</b>		<b>MARKS</b>
1.	CIE I	Mid Semester examination	15
2.	CIE II	Two Class Test of 15 marks each (Best of 2)	15
3.	CIE III	One Presentation/Seminar/ MCQ Test	10
4.	CIE IV	Class Assignments / One group discussion / Open Book Test	10
		<b>TOTAL</b>	<b>50</b>

**CIE for 4 credits Practical paper:** It will be divided as follows:

<b>SR. NO.</b>	<b>COMPONENTS</b>		<b>MARKS</b>
1.	CIE I	Mock Practical Examination	30
2.	CIE II	Viva Voce	10
3.	CIE III	Journal / project report/ dissertation report completion and certification on time.	10
		<b>TOTAL</b>	<b>50</b>

Above components will also be followed for 2 credit theory and practical paper.

**Syllabus:**

<b>Course/ Paper Title</b>	<b>Complex Analysis</b>
<b>Course Code</b>	<b>23SMMT31MM</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>The Complex Number System:</b>	<b>04</b>
	1. The field of Complex numbers.	1
	2. The complex plane.	1
	3. Polar representation and roots of complex numbers.	1
	4. The extended plane and its spherical representation.	1
<b>Unit II</b>	<b>Elementary Properties and Examples of Analytic Functions:</b>	<b>12</b>
	1. Power series.	4
	2. Analytic functions.	4
	3 Analytic functions as mappings, Möbius transformations	4
<b>Unit III</b>	<b>Complex Integration:</b>	<b>12</b>
	1. Riemann-Stieltjes integrals (without proof).	3
	2. Power series representation of analytic functions.	3
	3. Zeros of an analytic function.	3
	4. The index of a closed curve.	3
<b>Unit IV</b>	<b>Cauchy's Theorem:</b>	<b>12</b>
	1. Cauchy's Theorem and Integral Formula.	3
	2. The homotopic version of Cauchy's Theorem and simple connectivity (without proof of Theorem 6.7).	3
	3. Counting Zeros; the Open Mapping Theorem.	3
	4. Goursat's Theorem.	3

<b>Unit V</b>	<b>Singularities:</b>	<b>12</b>
	1. Classification of singularities.	4
	2. Residues.	4
	3. The Argument Principle.	4
<b>Unit VI</b>	<b>The Maximum Modulus Theorem:</b>	<b>08</b>
	1. The Maximum Principle.	4
	2. Schwarz's Lemma.	4

**Textbook:**

John B. Conway, Functions of One Complex Variable, 2<sup>nd</sup> Edition, Springer International Student Edition, Narosa Publishing House, 16<sup>th</sup> Reprint, 2002.

Unit I: Chapter 1: Sec. 2, 3, 4, 6.

Unit II: Chapter 3: Sec. 1, 2, 3.

Unit III: Chapter 4: Sec. 1, 2, 3, 4.

Unit IV: Chapter 4: Sec. 5, 6, 7, 8.

Unit V: Chapter 5: Sec. 1, 2, 3.

Unit VI: Chapter 6: Sec. 1, 2.

**Reference:**

**1. Books:**

1. Reinhold Remmert, Theory of Complex Functions, Springer, ISBN 9780387971957
2. Serge Lang, Complex Analysis, Fourth Edition, Springer, ISBN 9780387985923
3. Lars V. Ahlfors, Complex Analysis, Third Edition, McGraw-Hill, ISBN 0070850089
4. S. Ponnusamy, Herb Silverman, Complex Variables with Applications, Birkhauser Publications.

**2. Website:**

Complex Analysis - Pranav Haridas | Kerala School of Mathematics - NPTEL

[https://www.youtube.com/results?search\\_query=complex+analysis+professor+pranav+haridas](https://www.youtube.com/results?search_query=complex+analysis+professor+pranav+haridas)

<b>Course/ Paper Title</b>	<b>Field Theory</b>
<b>Course Code</b>	<b>23SMMT32MM</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No.</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Algebraic Extension of fields</b>	<b>20</b>
	1. Irreducible polynomials and Eisenstein criterion.	5
	2. Adjunction of roots.	5
	3. Algebraic extensions.	5
	4. Algebraically closed fields.	5
<b>Unit II</b>	<b>Normal and separable extensions</b>	<b>15</b>
	1. Splitting fields.	3
	2. Normal extensions.	3
	3. Multiple roots.	3
	4. Finite fields.	3
	5. Separable extension.	3
<b>Unit III</b>	<b>Galois Theory</b>	<b>10</b>
	1. Automorphism groups and fixed fields.	3
	2. Fundamental theorem of Galois theory.	4
	3. Fundamental theorem of algebra.	3
<b>Unit IV</b>	<b>Applications of Galois theory to classical problems</b>	<b>15</b>
	1. Roots of unity and cyclotomic polynomials.	3
	2. Cyclic extensions.	3

	3. Polynomials solvable by radicals.	3
	4. Symmetric functions.	3
	5. Ruler and compass constructions.	3

### **Textbook:**

P. B. Bhattacharyya, S. K. Jain, S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, Second Edition.

Unit I: Chapter 15.

Unit II: Chapter 16.

Unit III: Chapter 17.

Unit IV: Chapter 18.

### **References:**

#### **1. Books:**

1. D. Dummit, R. M. Foote, Abstract Algebra, 2<sup>nd</sup> Edition, Wiley Eastern Ltd.
2. T. A. Hungerford, Algebra, Graduate Texts in Mathematics, Vol. 73, Springer Verlag, 1980 (Indian Reprint 2004).
3. O. Zariski, P. Sammuel, Commutative Algebra, Vol. 1, Van Nostrand.
4. I. S. Luthar, I. B. S. Passi, Algebra, Vol. 4, Field Theory, Narosa Publishing House.
5. M. Artin, Algebra, Prentice Hall India, Second Edition.
6. Joseph Rotman, Galois Theory, Springer, 2<sup>nd</sup> Edition, ISBN-9780387985411.

#### **2. Website:**

1. <https://nptel.ac.in/courses/111/101/111101117/>
2. [https://www.youtube.com/watch?v=G\\_BNxjRrQYI&list=PLyqSpQzTE6M94LuHxxu4OrViX4K45oH7](https://www.youtube.com/watch?v=G_BNxjRrQYI&list=PLyqSpQzTE6M94LuHxxu4OrViX4K45oH7)

<b>Course/ Paper Title</b>	<b>Programming with Python (Theory)</b>
<b>Course Code</b>	<b>23SMMT33TMM</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>02</b>

<b>Unit No.</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Introduction to Python, Python Objects</b>	<b>02</b>
	<ol style="list-style-type: none"> <li>1. Features of Python: Easy; Type and Run; Syntax; Mixing; Dynamic Typing; Built in Object Types; Numerous Libraries and Tools.</li> <li>2. Chronology and Uses: Chronology; Uses.</li> <li>3. Installation of Anaconda.</li> <li>4. Basic Data Types Revisited: Fractions.</li> <li>5. Strings.</li> <li>6. Lists and Tuples: List; Tuples; Features of Tuples.</li> <li>7. Introduction to NumPy.</li> <li>8. Introduction to Pandas Data Structures: Series; DataFrame; Index Objects.</li> </ol>	
<b>Unit II</b>	<b>Conditional Statements</b>	<b>02</b>
	<ol style="list-style-type: none"> <li>1. if, if-else, and if-elif-else constructs.</li> <li>2. The if-elif-else Ladder.</li> <li>3. Logical Operators.</li> <li>4. The Ternary Operator</li> <li>5. The get Construct.</li> <li>6. Examples.</li> </ol>	
<b>Unit III</b>	<b>Looping</b>	<b>02</b>
	<ol style="list-style-type: none"> <li>1. While.</li> <li>2. Patterns.</li> <li>3. Nesting and Applications of Loops in Lists.</li> </ol>	

<b>Unit IV</b>	<b>Functions</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. Features of a functions: Modular Programming; Reusability of Code; Manageability.</li> <li>2. Basic Terminology: Name of Functions; Arguments; Return Value.</li> <li>3. Definition and Invocation: Working.</li> <li>4. Type of Functions: Advantage of Arguments.</li> <li>5. Implementing Search.</li> <li>6. Scope.</li> <li>7. Recursion: Rabbit Problem; Disadvantages of Using Recursion.</li> </ol>	
<b>Unit V</b>	<b>Iterations, Generators, and Comprehensions</b>	<b>02</b>
	<ol style="list-style-type: none"> <li>1. The Power of “For”.</li> <li>2. Iterators.</li> <li>3. Defining an Iterable Object.</li> <li>4. Generators.</li> <li>5. Comprehensions.</li> </ol>	
<b>Unit VI</b>	<b>File Handling</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. The File Handling Mechanism.</li> <li>2. The Open Function and File Access Modes.</li> <li>3. Python Functions for File Handling: The Essential Ones; The OS Methods; Miscellaneous Functions and File Attributes.</li> <li>4. Command Line Arguments.</li> <li>5. Implementation and Illustrations.</li> <li>6. Reading and Writing Data in Text Format.</li> <li>7. Binary Data Formats.</li> </ol>	
<b>Unit VII</b>	<b>Strings</b>	<b>02</b>
	<ol style="list-style-type: none"> <li>1. The Use of “For” and “While”.</li> <li>2. String Operators: The Concatenation Operator (+); The Replication Operator; The Membership Operator.</li> </ol>	

	<ol style="list-style-type: none"> <li>3. Functions for String Handling: len(); Capitalize(); find(); count; Endswith(); Encode; Decode; Miscellaneous Functions.</li> </ol>	
<b>Unit VIII</b>	<b>Introduction to Object Oriented Paradigm</b>	<b>02</b>
	<ol style="list-style-type: none"> <li>1. Creating New Types.</li> <li>2. Attributes and Functions: Attributes; Functions.</li> <li>3. Elements of Object- Oriented Programming: Class; Object; Encapsulation; Data Hiding; Inheritance; Polymorphism; Reusability.</li> </ol>	
<b>Unit IX</b>	<b>Classes and Objects</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. Defining a Class.</li> <li>2. Creating an Object.</li> <li>3. Scope of Data Members.</li> <li>4. Nesting.</li> <li>5. Constructor.</li> <li>6. Constructor Overloading.</li> <li>7. Destructors.</li> </ol>	
<b>Unit X</b>	<b>Inheritance</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. Introduction to Inheritance and Composition: Inheritance and Methods, Composition.</li> <li>2. Inheritance: Importance and Types: Need of Inheritance; Types of Inheritance.</li> <li>3. Methods: Bound Methods; Unbound Method; Methods are Callable Objects; The Importance and Usage of Super; Calling the Base Class Function Using Super.</li> <li>4. Search in Inheritance Tree.</li> <li>5. Class Interface and Abstract Classes.</li> </ol>	
<b>Unit XI</b>	<b>Operator Overloading</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. <code>_init_</code> Revisited: Overloading <code>_init_</code> (Sort of).</li> <li>2. Methods for Overloading Binary Operators.</li> </ol>	

	<ol style="list-style-type: none"> <li>3. Overloading the += Operator.</li> <li>4. Overloading the &gt; and &lt; Operators.</li> <li>5. Overloading the _boolEan_ Operators: Precedence of _bool_over _len_.</li> <li>6. Destructors.</li> </ol>	
<b>Unit XII</b>	<b>Exception Handling</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. Importance and Mechanism: An example of Try/Catch; Manually Raising Exception.</li> <li>2. Built In Exceptions in Python:</li> <li>3. The Process: Exception Handling: Try/Except; Raising Exceptions.</li> <li>4. Crafting User Defined Exceptions.</li> <li>5. An Example of Exception Handling.</li> </ol>	

**Textbook:**

1. H. Bhasin: Python Basics, MERCURY LEARNING AND INFORMATION Dulles, Virginia Boston, Massachusetts New Delhi.

Unit I: Chapter-1: 1.2, 1.4, 1.5. Chapter 2: 2.2 to 2.4, Chapter-18.

Unit II: Chapter-3: 3.2 to 3.7.

Unit III: Chapter-4: 4.2 to 4.4.

Unit IV: Chapter-5: 5.2, to 5.8.

Unit V: Chapter-6: 6.2 to 6.6.

Unit VI: Chapter-7: 7.1, to 7.6.

Unit VII: Chapter-8: 8.2, to 8.4.

Unit VIII: Chapter-9: 9.2, 9.3, 9.4.

Unit IX: Chapter-10: 10.1, to 10.8.

Unit X: Chapter-11: 11.1to 11.5.

Unit XI: Chapter-12: 12.2, to 12.8.

Unit XII: Chapter-13: 13.2, to 13.6.

2. Wes McKinney, Python for Data Analysis, Second Edition, Published by O'Reilly Media.

Unit I: Chapter-5: Section 5.1.

Unit VI: Chapter-6: Sections 6.1, 6.2.

### References:

#### 1. Books:

1. Mark Lutz, Programming Python, Publication O'reilly.
2. Wesley J. Chun, Core Python Programming, Publication Prentice Hall.
3. Python: Notes for Professionals, Goalkicker.com, Free Programming books.
4. Excel Formulas Bible- Excel 2013/2016.

#### 2. Website:

1. <https://www.tutorialsteacher.com/python>
2. <https://www.tutorialspoint.com/python/index.htm>

#### Best IDE TOOLS for Python:

Sr. No.	Tool	Version
1	Spyder	Version 5.2.1
2	Python 3.8.10	Python 3.8.10

<b>Course/ Paper Title</b>	<b>Programming with Python (Practical)</b>
<b>Course Code</b>	<b>23SMMT33PMM</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>02</b>

<b>Practical No.</b>	<b>Practical Name</b>	<b>No. of Hours</b>
1	Introduction to Python	4
2	Data types and Operators in python	4
3	Data types and Operators in python	4
4	Iterations and Conditional Statements	4
5	Iterations and Conditional Statements	4
6	Python Functions, Modules & Packages	4
7	Python Functions, Modules & Packages	4
8	Files and Directories	4
9	Practical on NUMPY Library	4
10	Practical on Pandas Library	4
11	Practical on MATPLOTLIB	4
12	Practical on Reading and Writing Data from .csv, .xlsx file.	4

\* Three practical sessions for CIE.

<b>Course/ Paper Title</b>	<b>Fourier Series and Boundary Value Problems</b>
<b>Course Code</b>	<b>23SMMT34MM</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>02</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Fourier Series</b>	<b>5</b>
	1. Piecewise Continuous Functions, Fourier Cosine Series, Examples.	1
	2. Fourier Sine Series, Examples.	1
	3. Fourier Series, Examples.	1
	4. Adaptations to Other Intervals.	2
<b>Unit II</b>	<b>Convergence of Fourier Series</b>	<b>5</b>
	1. One-Sided Derivatives, A Property of Fourier Coefficients.	1
	2. Two Lemmas, A Fourier Theorem, Discussion of the Theorem and its Corollary, Convergence on Other Intervals, Lemma.	2
	3. Absolute and Uniform Convergence of Fourier series, Differentiation of Fourier Series, Integration of Fourier Series.	2
<b>Unit III</b>	<b>The Fourier Method</b>	<b>04</b>
	1. Linear Operators, Principle of Superposition.	2
	2. A Temperature Problem, A Vibrating String Problem.	2

<b>Unit IV</b>	<b>Boundary Value Problems</b>	<b>6</b>
	1. A Slab with Faces at Prescribed Temperatures, Related Problems, A Slab with Internally Generated Heat, Steady Temperatures in a Rectangular Plate.	2
	2. Cylindrical Coordinates, String with Prescribed Initial Conditions, Resonance, Elastic Bar.	2
	3. Double Fourier Series, Periodic Boundary Conditions.	2
<b>Unit V</b>	<b>Orthonormal Sets</b>	<b>05</b>
	1. Inner Products and Orthonormal Sets, Examples.	1
	2. Generalized Fourier Series, Examples.	1
	3. Best Approximation in the Mean, Bessel's Inequality and Parseval's Equation.	2
	4. Application to Fourier Series.	1
<b>Unit VI</b>	<b>Sturm-Liouville Problems and Applications</b>	<b>5</b>
	1. Regular Sturm-Liouville Problems, Modifications, Orthogonality of Eigenfunctions, Real-Valued Eigenfunctions and Nonnegative Eigenvalues, Methods of solution.	2
	2. Examples of Eigenfunctions Expansions, A Temperature Problem in Rectangular Coordinates , Another Problem, Other Coordinates.	2
	3. Modification of the Method, Another Modification, A Vertically Hung Elastic Bar.	1

**Textbook:**

J.W. Brown, R.V. Churchill, Fourier Series and Boundary Value Problems, 7<sup>th</sup> Edition, McGraw Hill Education (India) Private Limited, New Delhi.

Unit I: Chapter 1.

Unit II: Chapter 2.

Unit III: Chapter 4.

Unit IV: Chapter 5.

Unit V: Chapter 7.

Unit VI: Chapter 8.

**References:**

**1. Books:**

1. Murray Spiegel, Fourier Analysis with Applications to Boundary Value Problems, Schaum's Outline Series, McGraw Hill.

**2. Website:**

1. [https://www.youtube.com/watch?v=HoGNkZclxDU&list=PLs7oDAL8\\_ouJ5w8wCPtKnK2I09MIKC6kP](https://www.youtube.com/watch?v=HoGNkZclxDU&list=PLs7oDAL8_ouJ5w8wCPtKnK2I09MIKC6kP)
2. <https://nptel.ac.in/courses/111/106/111106046/>

<b>Course/ Paper Title</b>	<b>Probability and Stochastic Process</b>
<b>Course Code</b>	<b>23SMMT31MEA</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Probability space</b>	<b>08</b>
	1. Introduction.	1
	2. Discrete probability space.	1
	3. Basic rules of probability.	1
	4. Conditional probability.	2
	5. Independence of events.	1
	6. Baye's formula	2
<b>Unit II</b>	<b>Discrete random variable</b>	<b>16</b>
	1. Definition.	1
	2. Probability distribution	1
	3. Expectation, variance, standard deviation of a discrete random variable.	2
	4. The Bernoulli random variable, the binomial random variable, the geometric random variable, the poisson random variable	4
	5. Joint probability distribution.	2
	6. Covariance and correlation coefficient.	1
	7. Independence of random variables.	2
	8. Conditional probability and conditional expectation.	3
<b>Unit III</b>	<b>Continuous random variables</b>	<b>15</b>

	<ol style="list-style-type: none"> <li>1. Definition.</li> <li>2. Probability distribution.</li> <li>3. Expectation, variance, standard deviation of a continuous random variable.</li> <li>4. The uniform random variable, exponential random variables, gamma random variables, normal random variables.</li> <li>5. Joint probability density and probability distribution.</li> <li>6. Covariance and correlation coefficient.</li> <li>7. Independence of random variables.</li> </ol>	<p>1</p> <p>2</p> <p>2</p> <p>4</p> <p>2</p> <p>2</p> <p>2</p>
<b>Unit IV</b>	<b>Central limit theorem</b>	<b>04</b>
	<ol style="list-style-type: none"> <li>1. Markov's and Chebyshev's inequality for discrete random variables and continuous random variables.</li> <li>2. Weak law of large numbers.</li> <li>3. Central limit theorem.</li> </ol>	<p>2</p> <p>1</p> <p>1</p>
<b>Unit V</b>	<b>Markov chains and countable state space</b>	<b>17</b>
	<ol style="list-style-type: none"> <li>1. Definition</li> <li>2. Examples.</li> <li>3. Gambler's ruin problem, Ehrenfest model, random walk on <math>Z, Z^2, Z^3</math> lattice.</li> <li>4. Hitting times and hitting probabilities.</li> <li>5. Recurrence and transience.</li> <li>6. States in a Markov chain</li> <li>7. Stationary measure and distribution.</li> <li>8. Convergence theorem for irreducible and aperiodic chains.</li> </ol>	<p>1</p> <p>2</p> <p>3</p> <p>1</p> <p>2</p> <p>2</p> <p>3</p> <p>3</p>

**Textbook:**

1. Sheldon H. Ross, Introduction to probability models, 9<sup>th</sup> edition, Academic Press.

Unit I: Chapter 1.

Unit II: Chapter 2, Chapter 3.

Unit III: Chapter 2 Chapter 3.

Unit IV: Chapter 2 Chapter 3.

2. Lecture notes of Prof. Manjunath Krishnapur.

Unit: V.

**References:****1. Books:**

1. Paul G. Hoel, Sidney C. Port, Charles .J. Stone, Introduction to probability theory.
2. Geoffrey R. Grimmet and David R. Stirzaker, Probability and random processes, 4<sup>th</sup> Edition.

<b>Course/ Paper Title</b>	<b>Mechanics</b>
<b>Course Code</b>	<b>23SMMT31MEB</b>
<b>Semester</b>	<b>III</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit –I</b>	<b>Lagrange’s Formulation</b>	<b>18</b>
	1. Equation of Motion and conservation Theorems, Equation of Motion of a Particle, Equation of Motion of a System of Particle.	6
	2. Conservation Theorem of Linear Momentum of the system of particles, Angular Momentum of the system of Particle, Constraint Motion, Examples of motion under constraints, Holonomic and Non – Holonomic Constraints, Scleronomic and Rheonomic Constraints, Degrees of Freedom and Generalized Co – ordinates.	6
	3. Transformation Relations, Virtual work, Principle of Virtual Work, D., Alembert’s Principle, Conservation of Energy, Kinetic Energy as a Homogeneous quadratic function of generalized velocities, Another way of proving conservation Theorem for Energy, Lagrange’s Equations for Non – holonomic Constraints.	6
<b>Unit II</b>	<b>Variational Principles</b>	<b>12</b>
	1. Generalization of Theorem, Minimum surface of revolution, Brachistochrone Problem, A case of variable	

	<p>end points along vertical lines <math>x = a</math> and <math>x = b</math> , Integrand as a function of more than two dependent variables.</p> <p>2. Isoperimetric problems, variational problems with moving boundaries.</p> <p>3. Functional dependent on functions of two dependent variables.</p>	<p>3</p> <p>3</p> <p>3</p>
<b>Unit III</b>	<b>Hamilton's Principle</b>	<b>18</b>
	<p>1. Hamilton's Principle for Non – Conservative and Conservative Systems, Configuration Space and Phase Space, Lagrange's Equations of Motion from Hamilton's Principle, Hamiltonian Formulation, Hamiltonian Function, Hamilton's Canonical Equations of Motion for partially.</p> <p>2. Conservative and Partially Non – Conservative System, Derivation of Hamilton's Equations of Motion from Hamilton's Principle, Physical Meaning of the Hamiltonian.</p> <p>3. Conservative and Scleronomic system, Non–conservative and Rheonomic system, partially conservative, Partially Non –conservative system, Cyclic co – ordinates in Hamiltonian, Routh's Procedure, Principle of Least Action.</p>	<p>6</p> <p>6</p> <p>6</p>
<b>Unit IV</b>	<b>Two Body Central Force Motion</b>	<b>12</b>
	<p>1. Reduction of Two body problem to an equivalent one Body problem, Equation of Motion and the First Integral.</p> <p>2. Kepler's Laws of Planetary Motion, Kepler's First second and Third Law, Deduction of Kepler's Laws, Escape velocity.</p> <p>3. Newton's law of Gravitation from Kepler's Laws of Planetary Motion, Differential Equation of the orbit of a Particle, Virial Theorem.</p>	<p>4</p> <p>4</p> <p>4</p>

**Textbook:**

**Problems in Classical Mechanics by L. N. Katkar (Narosa Publication)**

Chapter 1 To 4

**References:****1. Books:**

1. Classical Mechanics (3<sup>rd</sup> Ed.) **by** Herbert Goldstein, Charles Poole, John Safko (Pearson Education)
2. Classical Mechanics **by** Gupta, Kumar and Sharma (A Pragati Edition)]
3. Classical Mechanics **by** Rana Joag (Mc Graw Hill India)
4. Classical Mechanics **by** R. N. Tiwari and B. S. Thakur (PHI)

**2. Website:**

<https://www.youtube.com/watch?v=s2RmqPIfETc&list=PLdOM9VfSZfHNPdJCUthjWHBeAgyn3IuzM>

<b>Course/ Paper Title</b>	<b>Functional Analysis</b>
<b>Course Code</b>	<b>23SMMT41MM</b>
<b>Semester</b>	<b>IV</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Banach Spaces</b>	<b>25</b>
	1. The definition and some examples.	5
	2. Continuous linear transformations.	5
	3. The Hahn-Banach theorem.	5
	4. The natural imbedding of $N$ in $N^{**}$ .	5
	5. The open mapping theorem.	5
<b>Unit II</b>	<b>Hilbert Spaces</b>	<b>25</b>
	1. The definition and some simple properties.	3
	2. Orthogonal complements.	3
	3. Orthonormal sets.	3
	4. The conjugate space $H^*$ .	4
	5. The adjoint of an operator.	3
	6. Self-adjoint operators.	3
	7. Normal and unitary operators.	3
	8. Projections.	3

<b>Unit III</b>	<b>Finite-Dimensional Spectral Theory</b>	<b>10</b>
	1. Matrices.	2
	2. Determinants and the spectrum of an operator.	3
	3. The spectral theorem.	3
	4. A survey of the situation.	2

**Textbook:**

G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill.

Unit I: Chapter 9.

Unit II: Chapter 10.

Unit III: Chapter 11.

**References:**

**1. Books:**

1. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
2. George Bachman, Lawrence Narici, Functional Analysis, Dover Publications.
3. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley, 1989.
4. S. Kesavan Functional Analysis, Hindustan Book Agency(India), ISBN 978-81-85931-87-6, ISBN 978-93-86279-42-2(eBook).
5. H. L. Royden Real Analysis, Third Edition, Prentice-Hall of india Private Limited, ISBN-8120309731.

**2. Website:**

1. <https://nptel.ac.in/courses/111/105/111105037/>
2. <https://nptel.ac.in/courses/111/106/111106147/>

<b>Course/ Paper Title</b>	<b>Differential Geometry</b>
<b>Course Code</b>	<b>23SMMT42MM</b>
<b>Semester</b>	<b>IV</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>		<b>15</b>
	1. Graphs and Level Sets.	3
	2. Vector Fields.	4
	3. The Tangent Space	4
	4. Surfaces.	4
<b>Unit II</b>		<b>20</b>
	1. Vector Fields on Surfaces; Orientation.	5
	2. The Gauss Map.	5
	3. Geodesics.	5
	4. Parallel Transport.	5
<b>Unit III</b>		<b>15</b>
	1. The Weingarten Map.	5
	2. Curvature of Plane Curves.	5
	3. Arc Length and Line Integrals.	5
<b>Unit IV</b>		<b>10</b>
	1. Curvature of Surfaces	10

**Textbook:**

John A. Thorpe, Elementary Topics in Differential Geometry, First Indian Reprint, Springer Publication, ISBN 978-81-8128-144-9.

Unit I: Chapter 1 to 4.

Unit II: Chapter 5 to 8.

Unit III: Chapter 9 to 11.

Unit IV: Chapter 12.

**References:****1. Books:**

1. Erwin Kryszig, Differential Geometry, Dover Publications Inc.
2. Christian Bar, Elementary Differential Geometry, Cambridge University Press.
3. Andrew Pressley, Elementary Differential Geometry, Springer.
4. T.J. Willmore, An Introduction to Differential Geometry, Dover Publications Inc.

**2. Website:**

1. <https://nptel.ac.in/courses/111/104/111104095/>
2. <https://www.youtube.com/watch?v=W9eFFO6pxWw&list=PL4fpys7KOcYg7TixVqy3F4ehhDNk97ZqL>



<b>Unit IV</b>	<b>Handling large data on a single computer</b>	<b>04</b>
	<ol style="list-style-type: none"> <li>1. General techniques for handling large volumes of data.</li> <li>2. General programming tips for dealing with large data sets.</li> <li>3. Case study predicting malicious URLs.</li> </ol>	<p style="text-align: center;">2 1 1</p>
<b>Unit IV</b>	<b>Machine Learning Model accuracy metrics and errors</b>	<b>04</b>
	<ol style="list-style-type: none"> <li>1. Confusion matrix, F1-score, ROC</li> <li>2. MAPE, RMSE, accuracy</li> </ol>	<p style="text-align: center;">2 2</p>
<b>Unit V</b>	<b>Applications of Machine Learning algorithm</b>	<b>05</b>
	<ol style="list-style-type: none"> <li>1. Fraud detection</li> <li>2. Prediction of rainfall</li> </ol>	<p style="text-align: center;">2 3</p>
<b>Unit VI</b>	<b>Data visualization</b>	<b>03</b>
	<ol style="list-style-type: none"> <li>1. Data visualization with Excel. Box plot, bar graph, Histogram, pie chart, line graph.</li> </ol>	<p style="text-align: center;">03</p>

**Textbook:**

1. Davy Cielen, Arno D. B. Meysman, Mohamed Ali, Introducing Data Science, Manning Publications Co., 1st edition, 2016.

Unit I: Chapter 1: 1.1 to 1.4.

Unit II: Chapter 2: 2.1, 2.3 to 2.5.

Unit III: Chapter 3: 3.1 to 3.4.

Unit IV: Chapter 4: 4.2 to 4.4.

Unit V: Chapter 5: 5.1 to 5.2.

Unit VI: Chapter 8: 8.2, Chapter 9: 9.1.

2. Peters Morgan, Data Analysis from Scratch with Python, AI Sciences. ISBN-13: 978-1721942817.

Unit III: Chapters 10, 11, 12.

## References:

### 1. Books:

1. An Introduction to Statistical Learning: with Applications in R, Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani, Springer, 1st edition, 2013.
2. Introduction to Machine Learning, Ethem Alpaydin, Third Edition 2018 PHI Learning Private Limited.

### 2. Website:

1. [https://www.youtube.com/watch?v=fn1rKKNLuzk&list=PL15FRvx6P0OWTINBS\\_93NHG2hIn9cynVT](https://www.youtube.com/watch?v=fn1rKKNLuzk&list=PL15FRvx6P0OWTINBS_93NHG2hIn9cynVT)
2. [https://www.youtube.com/watch?v=4SJ7bEILPjk&list=PLLy\\_2iUCG87CNaffzNZPva9rW-QmOmEv](https://www.youtube.com/watch?v=4SJ7bEILPjk&list=PLLy_2iUCG87CNaffzNZPva9rW-QmOmEv)

<b>Course/ Paper Title</b>	<b>Introduction to Data Science (Practical)</b>
<b>Course Code</b>	<b>23SMMT43PMM</b>
<b>Semester</b>	<b>IV</b>
<b>No. of Credits</b>	<b>02</b>

<b>Practical No.</b>	<b>Practical Name</b>	<b>No. of Hours</b>
1	Reading xlsx files, reading csv file through python	4
2	Writing csv and xlsx files with pandas with given column names and dates	4
3	Practical on NUMPY Library : maths functions walk through	4
4	Practical on NUMPY Library	4
5	Data manipulation with PANDAS library with Data Frame	4
6	Data manipulation with PANDAS library	4
7	Practical on Data cleaning, Find Outlier with IQR method	4
8	Practical on Data preparation and visualization	4
9	Practical on preparation and visualization	4
10	Practical on Machine Learning Algorithm: Linear Regression	4
11	Practical on Machine Learning Algorithm: Decision Tree Classification	4
12	Practical on Machine Learning: Confusion matrix, F1-Score	4

\* Three practical sessions for CIE.

Open data set sites: <https://data.gov.in/>

<b>Course/ Paper Title</b>	<b>Number Theory</b>
<b>Course Code</b>	<b>23SMMT41MEA</b>
<b>Semester</b>	<b>IV</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No.</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Unique Factorization</b>	<b>12</b>
	1. Unique Factorization in $\mathbb{Z}$ , Unique Factorization in $k[x]$ .	4
	2. Unique Factorization in a Principal Ideal Domain.	4
	3. The Rings $\mathbb{Z}[i]$ and $\mathbb{Z}[\omega]$ .	4
<b>Unit II</b>	<b>Congruence</b>	<b>12</b>
	1. Congruence in $\mathbb{Z}$ , The congruence $ax \equiv b(m)$ .	6
	2. The Chinese Remainder Theorem.	6
<b>Unit III</b>	<b>Quadratic Reciprocity</b>	<b>12</b>
	1. Quadratic Residues.	6
	2. Quadratic Reciprocity.	6
<b>Unit IV</b>	<b>Some Functions of Number Theory</b>	<b>12</b>
	1. The Greatest Integer Function.	4
	2. Arithmetic Functions.	4
	3. The Möbius Inversion Formula.	4

<b>Unit V</b>	<b>Algebraic Numbers</b>	<b>12</b>
	1. Polynomials (Revision), Algebraic Numbers.	4
	2. Algebraic Number Fields, Algebraic Integers.	4
	3. Quadratic Fields, Units in Quadratic Fields. Primes in Quadratic Fields.	4

**Textbook:**

1) Kenneth Ireland, Michael Rosen, A Classical Introduction to Modern Number Theory, Springer, 4th Indian Reprint, 2013.

Unit I: Chapter 1: Arts 1 to 4.

Unit II: Chapter 3: Arts 1 to 4.

2) Ivan Niven, Herbert Zuckerman, Hugh Montgomery: An Introduction to Theory of Numbers, John Wiley and Sons, 5th Edition.

Unit III: Chapter 3: Arts 3.1 and 3.2.

Unit IV: Chapter 4: Arts 4.1 to 4.3.

Unit V: Chapter 9: Arts 9.1 to 9.7.

**References:**

**1. Books:**

1. G. Telang, Number Theory, Tata McGraw Hill.
2. M. B. Nathanson, Methods in Number Theory, GTM, Springer 3rd Indian Reprint, 2009.

**2. Website:**

1. <https://nptel.ac.in/courses/111/101/111101137/>
2. <https://www.youtube.com/watch?v=F659tUopJMg&list=PLR3C3NSCyhZQLANHMMCiSrTg6OJcSMpzD>

<b>Course/ Paper Title</b>	<b>Statistical Inference</b>
<b>Course Code</b>	<b>23SMMT41MEB</b>
<b>Semester</b>	<b>IV</b>
<b>No. of Credits</b>	<b>04</b>

<b>Unit No</b>	<b>Title with Contents</b>	<b>No. of Lectures</b>
<b>Unit I</b>	<b>Correlation and Regression Analysis</b>	<b>15</b>
	1. Introduction and Scatter Diagrams.	1
	2. Karl Pearson's Coefficient of Correlation.	2
	3. Spearman's Rank correlation coefficient.	1
	4. Method of Concurrent Deviations.	1
	5. Interpretation of r and Probable Error.	1
	6. Linear Regression.	2
	7. Lines of Regression.	1
	8. Theorems on Regression Coefficients.	2
	9. Yule's Rule.	1
	10. Order of Regression coefficients.	1
	11. Statistical Inferences about the Regression Parameters.	2
<b>Unit II</b>	<b>Hypothesis Testing</b>	<b>13</b>
	1. Statistical Hypothesis, general concepts	2
	2. Testing a statistical hypothesis	2
	3. Types of errors in testing of hypothesis	2
	4. Level of significance	1
	5. Critical regions	1
	6. Use of p values for decision making	1
	7. Tests of significance for single mean (variance known)	1
	8. Tests of significance for single mean (variance unknown)	1

	9. Confidence interval estimation, Probability distribution	2
<b>Unit III</b>	<b>Chi square distribution</b>	<b>13</b>
	1. Introduction of Chi-square Distribution	1
	2. Chi-square test for Goodness of Fit and its conditions for validity.	2
	3. Chi-Square test for independence of attributes.	2
	4. Degrees of Freedom.	2
	5. Test for equality of several Proportions.	2
	6. Chi-square test for population variance.	2
	7. Applications of Chi-square Distribution.	2
<b>Unit IV</b>	<b>Small Sample Tests</b>	<b>12</b>
	1. Critical Values and Applications of t-distribution.	2
	2. Confidence Interval for difference of two means.	2
	3. Paired t-test for difference of two Means.	2
	4. t-test for significance of an observed sample correlation coefficient.	2
	5. F-distribution and its applications.	1
	6. F-test for equality of population variances.	1
	7. Relation between t, F and Chi-square Distributions.	2
<b>Unit V</b>	<b>Analysis of Variance (ANNOVA)</b>	<b>07</b>
	1. One-way analysis of variance	1
	2. Two-way analysis of variance	2
	3. The Kruskal-Wallis one-way analysis of variance by ranks	2
	4. The Friedman two-way analysis of variance by ranks	2

**Textbook:**

1. Fundamentals of Statistics, by S. C. Gupta (Seventh Edition) (Chapters 8, 9, 13, 14, 16, 18, 19, 23)

**References:****1. Books:**

1. Introduction to Probability and Statistics for Engineers and Scientists, by Sheldon M. Ross (Fourth Edition).
2. Biostatistics, A Foundation for Analysis in Health Sciences, by Wayne W. Daniel (Eighth Edition, Wiley Publications).
3. Mathematical Statistics, by Parimal Mukhopadhyay.
4. Statistics for the Life Sciences, by M. Samules, J. Witmer and A. Schaffner (Fifth Edition, Pearson India).

**2. Website:**

1. <https://nptel.ac.in/courses/111/105/111105124/>
2. <https://nptel.ac.in/courses/111/105/111105043/>

# Semester III



**M. C. E. Society's**

**Abeda Inamdar Senior College**

Of Arts, Science and Commerce, Camp, Pune-1

(Autonomous) Affiliated to Savitribai Phule Pune University

NAAC accredited 'A' Grade

<b>Course/ Paper Title</b>	<b>Research Project</b>
<b>Course Code</b>	23SMMT31RP
<b>Semester</b>	III
<b>No. of Credits</b>	4 Credits

## Aims & Objectives of the Course

<b>Sr. No.</b>	<b>Objectives</b>
	Student should understand and learn;
<b>1.</b>	To maintain an updated curriculum.
<b>2.</b>	To take care of fast development in the knowledge of mathematics
<b>3.</b>	To enhance the quality and standards of Mathematics Education.
<b>4.</b>	To provide a broad common framework, for exchange, mobility, and free dialogue across the Indian Mathematical and associated community.

## Expected Course Specific Learning Outcomes

<b>Sr. No.</b>	<b>Learning Outcome</b>
<b>1.</b>	Students will have an aptitude to Study higher Mathematics and creative work in Mathematics.
<b>2.</b>	Students will equipped themselves with that part of Mathematics which is needed for various branches of Sciences or Humanities in which they have an aptitude for higher studies and original work.

## Details for 23SMMT31RP: Research Project

The candidates shall undertake the project work in the Third and Fourth Semester either in the Department concerned or in Industries, Research Institute, or any other Organizations (National / International).

In case the candidate undertakes the project work outside the Department, **the teacher concerned within the Department** shall be the Main co-guide and the teacher/scientist under whom the work is carried out will be the guide. The candidate shall bring the attendance certificate from the place where the project work carried out.

Following points must be included during project:

1. Students should undertake projects related to Mathematics.
2. Students should spend enough time for the project works (**more than 4** hours per week)
3. If student is performing project in another institute, for such a student, internal mentor will be allotted and he will be responsible for internal assessment of a student. In this case student has to obtain certificate from both external and internal mentor.
4. Systematic record of attendance of project students must be maintained by a mentor.
5. Daily Project work book / diary will be maintained by students and weekly checked from mentor / guide.
6. Students must present his monthly progressions to his mentor/guide/co-guide and this monthly progress report has to be submitted at time of examination.
7. Student is expected to do literature survey and review the selected research papers and find the application, if possible.
8. Project progression of third semester will be evaluated jointly by all examiners. Typically, student must present his review work and discuss difficulties in paper which will be followed by question-answer session (10 min). It is open type of examination.

# Semester IV



**M. C. E. Society's**

**Abeda Inamdar Senior College**

Of Arts, Science and Commerce, Camp, Pune-1

(Autonomous) Affiliated to Savitribai Phule Pune University

NAAC accredited 'A' Grade

<b>Course/ Paper Title</b>	<b>Research Project</b>
<b>Course Code</b>	23SMMT41RP
<b>Semester</b>	IV
<b>No. of Credits</b>	6 Credits

## Aims & Objectives of the Course

<b>Sr. No.</b>	<b>Objectives</b>
	Student should understand and learn;
<b>1.</b>	To maintain an updated curriculum.
<b>2.</b>	To take care of fast development in the knowledge of mathematics
<b>3.</b>	To enhance the quality and standards of Mathematics Education.
<b>4.</b>	To provide a broad common framework, for exchange, mobility, and free dialogue across the Indian Mathematical and associated community.

## Expected Course Specific Learning Outcomes

<b>Sr. No.</b>	<b>Learning Outcome</b>
<b>1.</b>	Students will have an aptitude to Study higher Mathematics and creative work in Mathematics.
<b>2.</b>	Students will equipped themselves with that part of Mathematics which is needed for various branches of Sciences or Humanities in which they have an aptitude for higher studies and original work.

## Details for 23SMMT41RP: Project

The candidates should continue the project work in the Fourth Semester as mentioned in course 23SMMT31RP.

Complete Project report copies must be submitted to department and guide at the time of examination. Project report must contain following point.

### Contents

**Project report must be written and submitted in a proper format as follows;**

- i. Certificate (Signed by Project guide and Head of the Department)
- ii. Certificates for Poster/Paper presented in conferences (if any)
- iii. Introduction (not more than 6 pages)
- iv. Give proper definitions, examples and state the theorems proved in the concerned paper.
- v. Give detailed proofs of the theorems cited.
- vi. Give references including textbooks used to prove the theorem in (v) above.
- vii. Other relevant research paper.
- x. Acknowledgement

Final Project will be evaluated jointly by all examiners. Typically, student must present his project work and discuss results and conclusions in details (15-20 min.) which will be followed by question-answer session (10 min). It is open type of examination.

## MODALITIES AND STUDIES AND REPORT MAKING

### A. For sem -III

1. Make a survey of the existing literature in the topic of the students choice. e.g. “**Labelling of graphs**”.
2. For a topic “**Labelling of graphs**” see e.g: the online website [www.combinatorics.org](http://www.combinatorics.org)
3. From the website located, go to “DYNAMIC SURVEYS”.
4. Search the topic “Graph Labeling” (Joseph A. Gallian)
  - a. An abstract appears
  - b. Go to pdf version (There are over 350 graph labelling any one of these could be assigned to a student )
  - c. e.g. in the “contents” go to “3. Variations of Graceful Labeling”.
  - d. Go to say 3.7 ”Cardinal Labellings”
  - e. Prepare a detailed report on Cardinal Labellings, giving examples, citing a few results and citing references.

### B. For sem-IV

Follow up on 4(e)

- i. Give proper definitions, examples and state the theorems proved in the concerned paper.
- ii. Give detailed proofs of the theorems cited.
- iii. Give references including textbooks used to prove the theorem in (ii) above.

Mrs. H.J. Siamwalla  
Chairman, BoS Mathematics  
and Head, Department of Mathematics